

q-deformed pairing-vibrations

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Abstract

Boson creation operators constructed from linear combinations of q- deformed zero coupled nucleon pair operators acting on the nucleus (A,0), are used to derive pp-RPA equations. The solutions of these equations are the pairing vibrations in $(A\pm 2)$ nuclei. For the 0_1^+ and 0_2^+ states of the nucleus ^{208}Pb , the variations of relative energies and transfer cross-sections for populating these states via (t,p) reaction, with deformation parameter τ have been analysed. For $\tau = 0.405$ the experimental excitation energy of 4.87MeV and the ratio $\frac{\sigma(0_2^+)}{\sigma(0_1^+)} = 0.45$ are well reproduced. The critical value of pairing interaction strength for which phase transition takes place, is seen to be lower for deformed zero-coupled nucleon pair condensate with τ real, supporting our earlier conclusion that the real deformation simulates the two-body residual interaction. For τ purely imaginary a stronger pairing interaction is required to bring about the phase transition. The effect of imaginary deformation is akin to that of an antipairing type repulsive interaction.

Using deformed zero coupled quasi-particle pairs, a deformed version of Quasi-boson approximation for 0^+ states in superconducting nuclei is developed. For the test model of 20 particles in two shells, the results of q-deformed boson and quasi-boson approximations have been compared with exact results. It is found that the deformation effectively takes into account the anharmonicities and may be taken as a quantitative measure of the correlations not being accounted for in a certain approximate treatment.

1 Introduction

The notion of quantum groups has attracted a lot of attention over the last few years. The quantization by deformation was studied earlier by Bayen et. al [1]. Drinfeld [2] generalized this idea to quantize classical Lie algebras so as to construct non-commutative Hopf algebra structure. In particular the quantum group $SU_q(2)$, the q-analog of $SU(2)$, has been extensively studied by Jimbo [3], Woronowicz [4] and Pasquier[5]. By constructing a q-analogue of the quantum harmonic oscillator, Biedenharn [6] and Macfarlane [7] have generalized to $SU_q(2)$, the Schwinger approach to the quantum theory of angular momentum. The q-deformed algebras have found various applications to physical situations in nuclear and molecular physics [8][9][10]. Raychev et al.[8] suggest that quantum algebra is appropriate for the description of stretching effects in rotational nuclei and have found good fits of rotational spectra of even-even rare earths and actinides by using a hamiltonian proportional to the second order Casimir operator of the quantum algebra $SU_q(2)$. The $SO_q(4)$ quantum algebra has also been used for the description of q-analogue of the hydrogen atom[11]. In our earlier work [12] we constructed a q-deformed analogue of zero coupled nucleon pair states and found these to be more strongly bound than the pairs with zero deformation, when a real valued q-parameter is used. An interesting extension to the multi-shell case, with deformed zero coupled pairs distributed over several single particle orbits showed that the deformation essentially simulates the effective residual interaction. Bonatsos [13] has also shown that the commutation relations of operators for zero coupled correlated fermion pairs in a single-j orbit can be satisfied up to first order correction by suitably defined q-bosons, onto which the fermion pair operators are mapped. Presently, we construct the excitation operators for pairing vibrations from deformed zero coupled nucleon pairs as well as deformed zero coupled quasi-particle pairs and analyze the critical point behaviour of nuclei as the deformation parameter takes real and imaginary values.

Pairing vibrations [14, 15, 16] are the collective vibrations of zero coupled fermion pairs around the Fermi surface. The natural framework for studying the pairing vibrations is RPA for nonsuperconducting nuclei and Quasi-particle-RPA for superconducting nuclei. In well known works on pairing vibrations the main concern has not been the reproduction of

experimental data but the physical content of the model. As such several calculations are available for test nuclei in addition to those for real nuclei. It is seen that as the interaction between the pairs becomes stronger a large number of zero coupled pairs are able to cross the Fermi surface resulting in a phase transition of the nucleus. At this point the RPA approximation breaks down. The critical point behavior of a nuclear system is determined to a large extent by how the interaction between the pairs is taken into account. Our object is to analyze the characteristics of pairing vibrations induced by boson operators with $J^\pi = 0^+$ constructed from deformed zero coupled fermion pairs and deformed zero coupled quasi-particle pairs in order to have a better insight into the physical content of the deformation parameter.

We have organised the paper as follows. In section 2, we construct bosons from deformed zero coupled fermion pairs, set up the RPA equations and obtain the dispersion relation the solution of which gives boson excitation energies for non-superconducting nuclei. The energies of 0^+ states of Pb isotopes are discussed in section 3 and the calculated ratio of two nucleon transfer cross-sections for populating the 0_2^+ and 0_1^+ states of ^{208}Pb is compared with the experimental data to understand the extent to which anharmonicities are simulated by the deformation. Section 4 deals with the construction of deformed quasi-boson operators and obtaining deformed QP-RPA(quasi-particle Random Phase approximation) equations. The formalism of sections 2 and 4 is applied in section 5 to the test case of $N = 20$ particles in two shells and the results for the energy of the lowest 0^+ state from deformed boson approximation and deformed quasi-boson approximation compared with the exact results. Conclusions are presented in section 6. The definitions of normalized deformed fermion pair creation operator and quasi-particle pair creation operators are given in Appendices A and B respectively.

2 Deformed pair-RPA equations

In Ref. [12] it has been shown that the creation operator Z_0 and the annihilation operator \bar{Z}_0 for a deformed zero coupled nucleon pair in a shell-model orbit j may be expressed in terms of the generators of quantum group $SU_q(2)$. We can write(appendix (A)),

$$Z_0 = \frac{S_+(q)}{\sqrt{\{\Omega\}_q}} \quad ; \quad \bar{Z}_0 = \frac{S_-(q)}{\sqrt{\{\Omega\}_q}} \quad (1)$$

$$S_0 = \frac{(n_{op} - \Omega)}{2} \quad (2)$$

where $(2j + 1) = N = 2\Omega$ and

$$\{x\}_q = \frac{(q^x - q^{-x})}{(q - q^{-1})} \quad (3)$$

In the following discussion $q = e^\tau$. The vacuum state for q-deformed pairs is defined through

$$\bar{Z}_0|0\rangle = 0 \quad (4)$$

For nuclei with no superconducting solution, we construct the boson creation operator that links the ground state of the nucleus $|A, 0\rangle$ to the $J^\pi = 0^+$ eigenstate ν of the $(A+2)$ system that is

$$R_+^\nu = \sum_m X_m^\nu \left(\frac{S_{m+}(q)}{\sqrt{\{\Omega_m\}_q}} \right) - \sum_i Y_i^\nu \left(\frac{S_{i+}(q)}{\sqrt{\{\Omega_i\}_q}} \right) \quad (5)$$

such that

$$|A + 2, \nu\rangle = R_+^\nu |A, 0\rangle ; R^\nu |A, 0\rangle = 0 \quad (6)$$

We use the indices mn(ij) to refer to single particle levels above(below) Fermi level. The equation of motion[17] for the operator R_+^ν is

$$\langle A, 0 | [\delta R^\nu, [H, R_+^\nu]] | A, 0\rangle = \hbar\omega_\nu \langle A, 0 | [\delta R^\nu, R_+^\nu] | A, 0\rangle \quad (7)$$

where $\hbar\omega_\nu = (E_\nu(A + 2) - E_0(A))$. The amplitudes for zero coupled pair transfer to orbit j_m and j_i are given by,

$$X_m^\nu = \langle A + 2, \nu | \frac{S_{m+}(q)}{\sqrt{\{\Omega_m\}_q}} | A, 0\rangle ; Y_i^\nu = \langle A + 2, \nu | \frac{S_{i+}(q)}{\sqrt{\{\Omega_i\}_q}} | A, 0\rangle \quad (8)$$

For the case of independent particles interacting via a pairing force the system Hamiltonian is

$$H = \sum_r \epsilon_r n_{op}^r + H_P, \quad (9)$$

where H_P the pairing interaction between deformed pairs is given by

$$H_P = -G \sum_{r,s} S_{r+}(q) S_{s-}(q) \quad (10)$$

G being the pairing interaction strength parameter. This is a very simple model in which interaction strength between pairs is assumed to be the same irrespective of the j-value of the orbit that the pairs occupy. For this simple form of the Hamiltonian, the RPA equations are readily obtained by using the quocommutation relations Eqs.(46) for evaluating the equation

of motion, Eq.(7). For the two kinds of possible variations, $\delta R^\nu = \frac{S_{n-(q)}}{\sqrt{\{\Omega_n\}_q}}$ and $\delta R^\nu = \frac{S_{j-(q)}}{\sqrt{\{\Omega_j\}_q}}$, we obtain the following set of equations for the nucleus (A+2),

$$(\hbar\omega_\nu - 2\epsilon_n)X_n^\nu = -G\sqrt{\{\Omega_n\}_q}\sum_m X_m^\nu\sqrt{\{\Omega_m\}_q} - G\sqrt{\{\Omega_n\}_q}\sum_i Y_i^\nu\sqrt{\{\Omega_i\}_q} \quad (11)$$

$$(\hbar\omega_\nu - 2\epsilon_j)Y_j^\nu = G\sqrt{\{\Omega_j\}_q}\sum_m X_m^\nu\sqrt{\{\Omega_m\}_q} + G\sqrt{\{\Omega_j\}_q}\sum_i Y_i^\nu\sqrt{\{\Omega_i\}_q} \quad (12)$$

with solutions

$$X_n^\nu = -\frac{N^\nu\sqrt{\{\Omega_n\}_q}}{(\hbar\omega_\nu - 2\epsilon_n)} \quad Y_j^\nu = \frac{N^\nu\sqrt{\{\Omega_j\}_q}}{(\hbar\omega_\nu - 2\epsilon_j)}. \quad (13)$$

N^ν defined as

$$N^\nu = G\sum_m X_m^\nu\sqrt{\{\Omega_m\}_q} + G\sum_i Y_i^\nu\sqrt{\{\Omega_i\}_q} \quad (14)$$

evaluated by using the normalization condition,

$$\sum_n |X_n^\nu|^2 - \sum_j |Y_j^\nu|^2 = 1 \quad (15)$$

is given by

$$N^\nu = \left[\sum_n \frac{\{\Omega_n\}_q}{(2\epsilon_n - \hbar\omega_\nu)^2} - \sum_j \frac{\{\Omega_j\}_q}{(2\epsilon_j - \hbar\omega_\nu)^2} \right]^{-\frac{1}{2}}. \quad (16)$$

The dispersion relation obtained by summing up the Eqs. (11) and (12)

$$\frac{1}{G} = \sum_n \frac{\{\Omega_n\}_q}{(2\epsilon_n - \hbar\omega_\nu)} - \sum_j \frac{\{\Omega_j\}_q}{(2\epsilon_j - \hbar\omega_\nu)} \quad (17)$$

readily yields a graphical solution. A similar equation is obtained for the eigenstates of the nucleus (A - 2) using a two-hole boson creation operator of the form,

$$R_+^\mu = \sum_m X_m^\mu \left(\frac{S_{m-(q)}}{\sqrt{\{\Omega_m\}_q}} \right) - \sum_i Y_i^\mu \left(\frac{S_{i-(q)}}{\sqrt{\{\Omega_i\}_q}} \right) \quad ; \quad [H, R_+^\mu] = \hbar\omega_\mu R_+^\mu \quad (18)$$

with the two-hole phonon states given by $|(A-2), \mu\rangle = R_+^\mu |A, 0\rangle$.

The excited 0^+ states of the nucleus $|A, 0\rangle$ are the two-phonon states

$$|\nu, \mu\rangle = R_+^\nu R_+^\mu |A, 0\rangle \quad (19)$$

with excitation energy

$$E(0^+) = \hbar\omega_\nu + \hbar\omega_\mu \quad (20)$$

The operator for two nucleon transfer is

$$F = \sum_m S_{m+}(q) + \sum_i S_{i+}(q) \quad (21)$$

The amplitudes for populating the ground state and the excited states of the nucleus $|A, 0\rangle$, via two nucleon transfer reactions are

$$\langle A, 0 | FR_+^\mu | A, 0 \rangle = \frac{N^\mu}{G} \quad (22)$$

and

$$\langle \nu, \mu' | FR_+^\mu | A, 0 \rangle = \delta_{\mu\mu'} \frac{N^\nu}{G} \quad (23)$$

respectively. Here μ refers to the lowest energy boson.

3 0^+ states of Pb Isotopes

Pb isotopes are a well known and much studied example of pairing vibrations in non-superconducting nuclei. Presently we concentrate on the 0^+ states of the nucleus ^{208}Pb . The main interest is to calculate the energy of the double pairing vibration (DPV) state for the neutron pair vibrations in ^{208}Pb which is experimentally known to lie at 4.87 MeV with respect to ground state and has been subject of investigation in various theoretical works on pairing vibrations. Bes and Broglia[16] using a simplified model consisting of like particles interacting via a pairing force with constant matrix element predicted an excited 0^+ state at 4.9 MeV. The calculated ratio between the matrix elements populating the first excited state and the ground state via a (t,p) reaction in this model is 1.3. Broglia and Riedel[20] further analysed the $^{206}\text{Pb}(t, p)^{208}\text{Pb}$ data[19] and showed that the linear pairing model produces only a qualitative agreement with the experimental data. Sorensen also [18] investigated the neutron pairing vibrations in ^{208}Pb using a pairing force hamiltonian with constant matrix elements. His model however uses the idea that the collective excitations of the system can be understood in terms of the interaction between a few collective bosons, each of which can be expanded in terms of two-fermion states. The hamiltonian is expressed in terms of the collective bosons and diagonalized in appropriate collective boson vector space to obtain the properties of the 0^+ states. In the case of 0^+ states of ^{208}Pb , the effect of including anharmonicities in this way is that though the excitation energy of 0_2^+ state is affected only slightly, the ratio $\frac{\sigma(0_2^+)}{\sigma(0_1^+)}$ becomes closer to the experimental value of 0.45. Sorensen [18] points out that the inclusion of anharmonic terms causes an increase in the (t,p) cross section for going to the 0_1^+ state of ^{208}Pb and a decrease in the cross section for going to 0_2^+ state. Figures (1a and 1b) show a plot of left hand side of Eq.(17) as a function of the pairing vibration state energy $E = \hbar\omega$ for two-neutron particle(hole) addition to the target nucleus

^{208}Pb . The model space consists of six hole levels and seven particle levels, the Fermi level being $2g_{\frac{9}{2}}$. Experimental single particle energies have been used in the calculation. In figure(1a), we have $q = e^\tau$ with the real valued parameter τ taking values 0.1, 0.2, 0.3 and 0.4 respectively. The solid line curve corresponds to undeformed zero coupled pair calculation. We note that as the deformation increases, the transition to superconducting phase is seen to occur at progressively smaller values of interaction strength, G_c . This result is consistent with our earlier[12] conclusion that the real deformation simulates attractive residual interaction between the nucleons. For a purely imaginary deformation parameter τ with values i0.1, i0.2, i0.3 and i0.4 an opposite effect comes into evidence in fig.(1b) where G_c is seen to become larger as the deformation is increased. The imaginary deformation τ effectively decreases the binding energy of the pair. The variation of G_c versus $|\tau|$ has been plotted in figure (2) for real as well as imaginary values of τ .

As in earlier linear calculations [16][18] the energy of this state is well reproduced for $G=0.087$ in figure (1a). We notice however that in the same figure, energy eigen value of 4.87 MeV is reproduced for DPV also by using deformed pairs with different values of deformation parameter and corresponding G-values. To choose the deformation parameter that correctly simulates the anharmonicities, we examine the behaviour of two-neutron transfer amplitude F versus boson energy for populating the ground state 0_1^+ and the DPV state 0_2^+ of ^{208}Pb for different values of deformation parameter in figures (3a, and 3b). For phonon excitation energies close to the unperturbed energy, the F value is almost independent of the value of parameter τ for two nucleon addition to 0_2^+ . However in the region close to breakdown F is very sensitive to deformation. We may observe that in general for a given value of boson excitation energy a real valued deformation causes the (t,p) cross section for going to 0_1^+ state to increase and the cross section for going to 0_2^+ state to decrease. These results are very similar to those from more complex calculations of Sorensen[18] showing that the inclusion of anharmonicities results in an increase in the calculated value of $\sigma(0_1^+)$ and a decrease in calculated $\sigma(0_2^+)$. An opposite trend is seen in figure (3b) for imaginary values of τ . Figure (4) is a plot of the ratio $\frac{\sigma(0_2^+)}{\sigma(0_1^+)}$ versus $|\tau|$ (0_2^+ being the calculated DPV with energy 4.87 MeV). For $\tau = 0.405$ the experimental value[19] of $\frac{\sigma(0_2^+)}{\sigma(0_1^+)} = 0.45$ is well reproduced.

4 Superconducting nuclei

For superconducting nuclei, the operators for creating and destroying a zero coupled quasi-particle pair and the commutation relations satisfied by these (in quasi-boson approximation) are given in Eq.(52) of appendix B.

In analogy with the case of nonsuperconducting nuclei we expect that the effect of residual interaction between quasi-particles may be simulated by deformation of quasi-particle pairs. As such, we define the generators of $SU_q(2)$ for quasi-particle pairs satisfying the q-commutation relations by

$$[\mathcal{S}_{i-}(q), \mathcal{S}_{j+}(q)] = \{\Omega_i - \mathcal{N}_i\}_q \delta_{i,j} \quad (24)$$

$$\mathcal{N}^i \mathcal{S}_{i+}(q) = 2\mathcal{S}_{i+}(q) \quad ; \quad \mathcal{N}^i \mathcal{S}_{i-}(q) = -2\mathcal{S}_{i-}(q). \quad (25)$$

The creation operator for normalized quasi-particle pair state for a given j-shell is defined by

$$\mathcal{Z}^\dagger(q)|0\rangle = \frac{\mathcal{S}_+(q)}{\sqrt{\{\Omega\}_q}}|0\rangle. \quad (26)$$

Next we construct the quasi-boson creation operator as

$$Q_\nu^\dagger = \sum_p \left(X_p^\nu \mathcal{Z}_p^\dagger(q) - Y_p^\nu \mathcal{Z}_p(q) \right) \quad (27)$$

The ground state of the nucleus $|A, 0\rangle$ and the eigenstate ν of the nucleus $|A + 2, \nu\rangle$ are related by

$$|A + 2, \nu\rangle = Q_\nu^\dagger |A, 0\rangle \quad ; \quad Q_\nu |A, 0\rangle = 0 \quad (28)$$

To derive the equations satisfied by amplitudes X_p^ν 's and Y_p^ν 's we compute the commutators $[\delta Q_\nu, [\mathcal{H}_{11} + \mathcal{H}_c, Q_\nu^\dagger]]$ for the variations $\delta Q_\nu = \mathcal{Z}_m(q)$ and $\delta Q_\nu = \mathcal{Z}_m^\dagger(q)$ and take their expectation values with respect to the ground state $|A, 0\rangle$ i.e. evaluate,

$$\langle A, 0 | [\delta Q_\nu, [\mathcal{H}_c, Q_\nu^\dagger]] | A, 0 \rangle = \hbar \omega_\nu \langle A, 0 | [\delta Q_\nu, Q_\nu^\dagger] | A, 0 \rangle. \quad (29)$$

Here the Hamiltonian $\mathcal{H}_{11} + \mathcal{H}_c$ is obtained by replacing the normal quasi-particle pair creation and destruction operators in $H_{11} + H_c$ of appendix B by deformed quasi-particle pair creation and destruction operators that is

$$\mathcal{H}_{11} = \sum_i E_i \mathcal{N}^i \quad (30)$$

$$\begin{aligned} \mathcal{H}_c = & -G \left(\sum_{ij} u_i^2 u_j^2 \mathcal{S}_{i+}(q) \mathcal{S}_{j-}(q) - u_i^2 v_j^2 \mathcal{S}_{i+}(q) \mathcal{S}_{j+}(q) \right. \\ & \left. - v_i^2 u_j^2 \mathcal{S}_{i-}(q) \mathcal{S}_{j-}(q) + v_i^2 v_j^2 \mathcal{S}_{i-}(q) \mathcal{S}_{j+}(q) \right). \end{aligned} \quad (31)$$

In deformed quasi-boson approximation, on evaluating Eq.(29) the amplitudes X^ν 's and Y^ν 's are found to satisfy the following set of coupled equations,

$$(\hbar\omega_\nu - 2E_m)X_m^\nu = -G\sqrt{\{\Omega_m\}_q} \sum_p \sqrt{\{\Omega_p\}_q} \left(X_p^\nu (u_m^2 u_p^2 + v_m^2 v_p^2) - Y_p^\nu (u_m^2 v_p^2 + v_m^2 u_p^2) \right) \quad (32)$$

$$(\hbar\omega_\nu + 2E_m)Y_m^\nu = G\sqrt{\{\Omega_m\}_q} \sum_p \sqrt{\{\Omega_p\}_q} \left(Y_p^\nu (u_m^2 u_p^2 + v_m^2 v_p^2) - X_p^\nu (u_m^2 v_p^2 + v_m^2 u_p^2) \right). \quad (33)$$

The set of Eqs. (32,33) can be solved by using standard methods to furnish the roots $E = \hbar\omega_\nu$.

5 Two Shell Test Model

For analyzing the behaviour of pairing vibration states in the deformed boson approximation and deformed quasi-boson approximation, we examine these states for a test model in which $N = 2\Omega$ particles are distributed over two shell model orbits, each with a degeneracy 2Ω . The Shell model orbits have single particle energies given by $\frac{\epsilon}{2}$ and $-\frac{\epsilon}{2}$ respectively. Following Hogaasen [15], we plot in fig.(5) the lowest energy root in units of 2ϵ as a function of $\frac{G\Omega}{2\epsilon}$. The plot is for $N=20$ and shows the results for various values of deformation parameter τ along with the results for exact calculation, boson approximation with zero deformation ($\tau = 0.0$) and quasi-boson approximation without deformation. All boson approximation curves start at $\frac{E}{2\epsilon} = 1.0$ and terminate at $\frac{E}{2\epsilon} = 0.0$ for a G value characteristic of each deformation value. The deformed quasi-boson approximation curves in general follow the trend of the similar curve for $\tau = 0.0$. In fig.(5a) the deformation parameter takes the real values $\tau = 0.1, 0.15$ and 0.2 . We may notice that for a given value of pairing interaction strength G the deformation causes the energy values to be lowered as compared to the boson approximation results for $G < G_c$ and quasi-boson approximation results for $G > G_c$. It implies that a boson or a quasi-boson pair with real deformation is more strongly bound than its undeformed counterpart. As the quasi-boson approximation energies lie higher than the exact energy eigen value for a given G , by using a suitable deformation exact energies can be reproduced in the deformed quasi-boson approximation. In the case at hand, for $\tau = 0.15$ there is a good agreement between the deformed quasi-boson approximation calculation and

the exact result for values of G not very close to G_c . For deformed boson calculation the critical value of pairing strength G , for which the energy of the lowest energy state approaches zero, is seen to go up as the deformation is increased. In other words, increasing the deformation parameter causes the transition to superconducting phase to occur at successively lower values of G .

In fig.(5b) the deformation parameter is taken to be purely imaginary with the curves shown for $\tau = i0.05, i0.1$ and $i0.15$. For a given value of G , an imaginary deformation parameter τ raises the excitation energy in the boson as well as the quasi-boson approximation. It may be interpreted as an anti-pairing effect. As the boson approximation energies lie lower than the exact energy eigenvalues for the same G value, for a suitable choice of τ we can obtain a deformed boson approximation curve having a good overlap with the exact results for a reasonably large range of G values in the region away from the phase transition region. Deformation in this case simulates the correlations not accounted for in the boson approximation. In a realistic case, an imaginary deformation may be used as a measure of correlations caused by a residual repulsive interaction not being accounted for in the boson or quasi-boson approximation.

We have plotted in Fig.(5c) the deformed boson approximation energies for $\tau = i0.104$ and quasi-boson energies for $\tau = 0.15$ the deformation values for which a good agreement with the exact calculation results is obtained in a wide region away from the phase transition region.

6 Conclusions

We have constructed q-analogues of boson approximation and quasi-boson approximation to understand the physical meaning of real and imaginary deformation in the context of interaction between zero coupled pairs. The formalism for non-superconducting nuclei has been applied to the classic example of the Double Pairing Vibrational state 0_2^+ in the nucleus ^{208}Pb . For a real deformation value of $\tau = 0.405$, the calculation reproduces the observed excitation energy of the 0_2^+ state, $E = 4.87$ MeV, as well as the ratio of cross sections, $R = \frac{\sigma(0_2^+)}{\sigma(0_1^+)} = 0.45$, for populating the 0_2^+ and 0_1^+ states via (t,p) reaction. A linear calculation [16] without deformation produces a large value of $R=1.3$. For obtaining a better agreement with experiment a more complex calculation that takes into account anharmonic effects as of Ref[18] is necessary. However presently a good agreement with the experiment is obtained

in a much simpler model by using deformed pairs. We may infer that a real deformation of zero coupled pairs takes into account the anharmonicities not being accounted for in boson approximation in the simple model at hand with the same pairing strength for all orbits . We also note that as the deformation parameter is increased the transition to superconducting phase occurs for successively smaller values of pairing interaction strength parameter for τ real and for successively larger values of G for imaginary values of τ . This behaviour is consistent with our earlier conclusion[12] that the deformed fermion pairs are more strongly bound for real valued τ and the deformation in this case simulates the residual attractive interaction. Similar results have been obtained by Bonatsos et. al [21] in the framework of Moszkowski model where increasing q-deformation has been shown to facilitate the phase transition from the vibrational to the rotational behavior. For $q = e^{i\tau}$, on the other hand, deforming the pair amounts to weakening the pair. The role of complex valued q is akin to a pair breaking residual repulsive force.

In both the cases the collectivity of the state is seen to be a function of deformation parameter as is evident from the variation of calculated two nucleon transfer cross sections in figures(3a,3b). We may point out that deforming the zero coupled pairs is not the same as changing the value of the pairing strength parameter G . For phonon excitation energies close to the unperturbed energies the collectivity is almost independent of the value of parameter τ . However in the region close to phase transition it is very sensitive to deformation. The deformed pair Hamiltonian apparently accounts for many-body correlations, the strength of higher order force terms being determined by the deformation parameter .

For the test model of 20 particles in two shells, the results of q-deformed boson and quasi-boson approximations have been compared with exact results. The deformed boson approximation results for $\tau = i0.104$ and deformed quasi-boson energies for $\tau = 0.15$ overlap the exact calculation results in a wide region away from the phase transition region. Apparently the deformation effectively takes into account the correlations not being accounted for in approximate treatments. As such, in a realistic calculation the deformation parameter may be taken as a quantitative measure of correlations left over in an approximate treatment as compared to the exact treatment. The test model results that the deformed boson pairs and deformed quasi-boson pairs are more(less) strongly bound as compared to their undeformed counterparts, for real(imaginary) τ , confirms our earlier conclusions.

We may conclude that, in a realistic case, a real(imaginary) deformation may be used as a measure of anharmonicities caused by a residual attractive(repulsive) interaction not

being accounted for or a measure of correlations left over in an approximate treatment.

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A Normalized Deformed pair creation operator

We can write a zero-coupled pair for two nucleons [22] in a shell model orbit j as

$$Z_0 = -\frac{1}{\sqrt{2}} (A^j \times A^j)^0 \quad (34)$$

with

$$\overline{Z}_0 = \frac{1}{\sqrt{2}} (B^j \times B^j)^0 \quad (35)$$

where

$$A_{jm} = a_{jm}^\dagger \quad ; \quad B_{jm} = (-1)^{m+j} a_{j,-m} \quad (36)$$

The fermion creation and destruction operators a_{jm}^\dagger and a_{jm} satisfy the usual commutation relations

$$[a_{jm}^\dagger, a_{jm}] = 1 \quad (37)$$

And the number operator is defined as

$$n_{op} = \sum_m a_{jm}^\dagger a_{jm} \quad (38)$$

We can easily verify that

$$[Z_0, \overline{Z}_0] = \frac{n_{op}}{\Omega} - 1 \quad (39)$$

$$[n_{op}, Z_0] = 2 Z_0 \quad [n_{op}, \overline{Z}_0] = -2 \overline{Z}_0. \quad (40)$$

Here $(2j + 1) = N = 2\Omega$. We may rewrite our pair operators in terms of the well known quasi-spin operators by identifying,

$$S_+ = \sqrt{\Omega} Z_0 \quad S_- = \sqrt{\Omega} \overline{Z}_0 \quad (41)$$

$$S_0 = \frac{(n_{op} - \Omega)}{2}. \quad (42)$$

S_+ , S_- and S_0 are the generators of Lie algebra of SU(2) and satisfy the same commutation relations as the angular momentum operators.

$$[S_+, S_-] = 2S_0 \quad [S_0, S_\pm] = \pm S_\pm \quad (43)$$

Total quasi-spin operator is given by

$$S^2 = S_+ S_- + S_0(S_0 - 1) \quad (44)$$

An equivalent description of the state $|n, v\rangle$ can be given in terms of the total quasi-spin quantum number s (related to the seniority quantum number v through $s = \frac{(\Omega-v)}{2}$) and the eigenvalue of operator S_0 . The states $|s, s_0\rangle$ satisfy the following relations,

$$S^2 |s, s_0\rangle = s(s+1) |s, s_0\rangle \quad ; \quad S_0 |s, s_0\rangle = s_0 |s, s_0\rangle \quad (45)$$

We may now define q-deformed pairs in terms of the generators of $SU_q(2)$ satisfying the commutation relations,

$$[S_+(q), S_-(q)] = \{2S_0(q)\}_q \quad [S_0(q), S_{\pm}(q)] = \pm S_{\pm}(q) \quad (46)$$

where

$$\{x\}_q = \frac{(q^x - q^{-x})}{(q - q^{-1})} \quad (47)$$

Normalized two nucleon state in terms of the deformed pair creation operator is

$$Z_0|0\rangle = N S_+(q) = \frac{S_+(q)}{\sqrt{\{\Omega\}_q}} |0\rangle \quad (48)$$

As such the operator for the creation of a normalized deformed pair state is $\frac{S_+(q)}{\sqrt{\{\Omega\}_q}}$.

B Quasi-particle pair creation and annihilation operators

The creation and destruction operators for quasi-particles are defined in terms of fermion creation and destruction operators by

$$\alpha_{jm}^\dagger = u_j a_{jm}^\dagger - (-1)^{j-m} v_j a_{j-m} \quad ; \quad \alpha_{jm} = u_j a_{j-m} - v_j a_{jm}^\dagger \quad ; \quad u_j^2 + v_j^2 = 1 \quad (49)$$

Quasi-spin operators for quasi-particles in an orbit j ,

$$\mathcal{S}_+ = \sum_{m>0} (-1)^{j-m} \alpha_{jm}^\dagger \alpha_{j-m}^\dagger \quad ; \quad \mathcal{S}_- = \sum_{m>0} (-1)^{j-m} \alpha_{j-m} \alpha_{jm} \quad (50)$$

and the quasi-particle number operator defined as

$$\mathcal{N} = \sum_m \alpha_{jm}^\dagger \alpha_{jm} \quad (51)$$

satisfy the following commutation relations

$$[\mathcal{S}_{i-}, \mathcal{S}_{j+}] = (\Omega_i - \mathcal{N}^i) \delta_{ij} \quad ; \quad [\mathcal{N}^i, \mathcal{S}_{j\pm}] = \pm 2\mathcal{S}_{i\pm} \delta_{ij} \quad (52)$$

Normalized zero coupled quasi-particle pair creation operator for a given orbit j is given by,

$$Z^\dagger |0\rangle = \frac{S_+}{\sqrt{\{\Omega\}}} |0\rangle \quad (53)$$

The pairing Hamiltonian in terms of these operators is written as

$$H = H_{00} + H_{11} + H_{20} + H_{02} + H_c + H_{res} \quad (54)$$

Where

$$\begin{aligned} H_{00} &= \sum_i (\epsilon_i - \lambda) 2v_i^2 \Omega_i + G \left(\sum_i \Omega_i u_i v_i \right)^2 \\ H_{11} &= \sum_i [(\epsilon_i - \lambda)(u_i^2 - v_i^2) + 2G u_i v_i \left(\sum_j \Omega_j u_j v_j \right)^2] \mathcal{N}_i \\ H_{20} + H_{02} &= \sum_i [(\epsilon_i - \lambda) 2u_i v_i - G \sum_j \Omega_j u_j v_j (u_i^2 - v_i^2)] (\mathcal{S}_{i+} + \mathcal{S}_{i-}) \\ H_c &= -G \sum_{ij} (u_i^2 u_j^2 \mathcal{S}_{i+} \mathcal{S}_{j-} - u_i^2 v_j^2 \mathcal{S}_{i+} \mathcal{S}_{j+} \\ &\quad - v_i^2 u_j^2 \mathcal{S}_{i-} \mathcal{S}_{j-} + v_i^2 v_j^2 \mathcal{S}_{i-} \mathcal{S}_{j+}) \\ H_{res} &= G \sum_{ij} u_i v_i [\mathcal{N}_i (u_j^2 \mathcal{S}_{j-} - v_j^2 \mathcal{S}_{j+}) + (u_j^2 \mathcal{S}_{j+} - v_j^2 \mathcal{S}_{j-}) \mathcal{N}_i] \\ &\quad - G \sum_{ij} u_i v_i u_j v_j \mathcal{N}_i \mathcal{N}_j. \end{aligned} \quad (55)$$

Imposing the condition that H_{02} and H_{20} cancel each other and the vacuum state for quasi-particles represent the ground state of the nucleus with n nucleons, we obtain the well known expressions for occupancies and the gap parameter.

$$\begin{aligned} u_i^2 &= \frac{1}{2} \left(1 + \frac{\epsilon_i - \lambda}{E_i} \right) \quad ; \quad v_i^2 = \frac{1}{2} \left(1 - \frac{\epsilon_i - \lambda}{E_i} \right) \\ \sum_i \frac{\Omega_i}{E_i} &= \frac{2}{G} \quad ; \quad \sum_i \Omega_i \left(1 - \frac{\epsilon_i - \lambda}{E_i} \right) = n \\ E_i &= \sqrt{\Delta^2 + (\epsilon_i - \lambda)^2} \\ \Delta &= G \sum_i \Omega_i u_i v_i \end{aligned} \quad (56)$$

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Figure Captions

Fig. (1a) - A plot of left hand side of equation (17) as a function of $E = \hbar\omega$ for $q = e^\tau$ with τ real. The vertical straight lines are $2\epsilon_{3p_{\frac{1}{2}}} = -2.7$ MeV, $2\epsilon_{3g_{\frac{9}{2}}} = 4.15$ MeV, $\hbar\omega_\nu = -2.2$ MeV and $\hbar\omega_\mu = 2.65$ MeV.

Fig. (1b) - As in (1a) for τ imaginary.

Fig. (2) - The breakdown point value of pairing interaction strength G_C as a function of $|\tau|$ for τ real as well as imaginary.

Fig. (3a) - Two nucleon transfer amplitude for populating the states 0_1^+ and 0_2^+ of ^{208}Pb as a function of E for τ real.

Fig. (3b) - As in (3a) for τ imaginary.

Fig. (4) - The ratio of cross-sections for populating the states 0_2^+ and 0_1^+ via two-neutron transfer in ^{208}Pb , $\frac{\sigma(0_2^+)}{\sigma(0_1^+)}$ versus $|\tau|$ (0_2^+ is the calculated DPV state with energy 4.87 MeV) for τ real as well as imaginary.

Fig. (5a) - The lowest energy root in units of 2ϵ plotted as a function of $\frac{G\Omega}{2\epsilon}$. The plot for $N=20$ shows the results for deformed boson and quasi-boson approximations for τ real along with the exact calculation, boson approximation and quasi-boson approximation without deformation.

Fig. (5b) - As in (5a) for τ imaginary.

Fig. (5c) - As in (5a) for $\tau = i0.104$ in deformed boson approximation and for $\tau = 0.15$ in deformed quasi-boson approximation.

Keyword Abstract: Quantum group $SU_q(2)$, Pairing-interaction, q-deformed RPA and QRPA, 0^+ States of ^{208}Pb , Two neutron transfer cross-section, Test model of 20 particles in two shells